national accelerator laboratory

TM-458
2623.000

15-FT HBC OPTICS DESCRIPTION

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December 14, 1973

DATA AVAILABLE

a) A set of 104 fiducials is drawn on the walls of the chamber. A survey has been made of those fiducials (see W. Smart for details).

b) Various sets of measured pictures coming from the three hadron cameras. Averaging measurements give rise to a measurement error of the order of one to a few microns onto the film.

AXIS SYSTEMS DEFINITION

a) Chamber System.

Origin is near the center of the chamber, z-axis is vertical, positive up, and x-axis is about along the beam, negative toward the chamber's nose. The definition of that system comes from the fiducial survey.

b) Camera System.

Origin is entrance pupil, z-axis is normal to the film plane, positive toward it, x-axis is about the film path. Each camera system is related to the chamber system by a set of 6 parameters $X_c, Y_c, Z_c, \theta_1, \theta_2, \theta_3$. (See Appendix III.)

c) Film System.

This is the system where the measurement is made. It is related to the camera system by 2 parameters x_ω and y_ω, the coördinates of the optical axis in the film system.

After fitting the optics, that film system will be redefined by requesting that x_ω and y_ω be zero. (One can also request that θ_3 be zero.)

OPTICS DESCRIPTION

In the camera system a point X, Y, Z in space and its image x, y are related by:

$$x = \frac{D_1 X}{(Z - D_2)} \quad y = \frac{D_1 Y}{(Z - D_2)} \tag{1}$$

$$D_1 \text{ and } D_2 \text{ being functions of } r^2 = x^2 + y^2. \tag{2}$$

Equation (1) implies that an optical axis exists, all light rays intersecting it. Equation (2) implies that this axis is a cylindrical-symmetric axis.

CAMERA DISTORTIONS

As the preceding may not be true, it is assumed that x and y are related to the measured quantities x_m and y_m through a mapping $x = f(x_m, y_m)$, $y = g(x_m, y_m)$ which transforms a real film plane into an ideal one where Eqs. (1) and (2) hold.

OPTICS' POLYNOMIAL EXPANSION

The functions D_1, D_2, f, and g may be expanded in a lot of different ways. We will try to satisfy the three following conditions:

1. The various parameters have an understandable meaning.

2. Those parameters being found by a fit over a set of measurements, it would be nice to have them not correlated.

3. No parameter will have the same effect, at the first-order level, than any other parameter; no singular or quasi-singular matrix in the fit.

Satisfying condition (2).

$$\chi^2 = \Sigma\, [f - x(X, Y, Z)]^2 + \ldots \text{ with } f = \Sigma \lambda_i f_i(x_m, y_m).$$

To have no correlations needs that: $\Sigma\,(f_i f_j + g_i g_j) \propto \delta_{ij}$, or in other words that the set of 2-vectors $\vec{f} = \left|\begin{matrix} f_i \\ g_i \end{matrix}\right.$ has to be orthogonal. The metric is given by the set of measurements itself.

$$\int \vec{f}_i \vec{f}_j m(r, \theta)\, r dr d\theta = \delta_{ij} \quad \text{with } x_m = r\cos\theta$$
$$y_m = r\sin\theta$$

if m is a constant one gets some Bessel functions which are too time consuming to use. So we will give up the complete orthogonality and write down:

$$\vec{f} = \Sigma\, \vec{F}_n(r) \cos n\theta + \vec{G}_n(r) \sin n\theta\,.$$

$\vec{E}_n$ and $\vec{G}_n$ are orthogonal to one another; each one will be expanded in power of r the coefficients of each expansion being correlated among themselves.

Satisfying condition (1).

One has already gotten some insight with the F and G because we have now a classification according to the kind of symmetry. Another idea is to split radial and tangential components; this is because they have different meanings as far as optic laws are concerned and because it is easy to split the χ^2 in its radial and tangential components and thus see better the effect of the various parameters tried. So we will write down:

$$x = R x_m - T y_m + \Sigma\, a_n r^n \cos n\theta + b_n r^n \sin n\theta$$

$$y = R y_m + T x_m + \Sigma\, b_n r^n \cos n\theta - a_n r^n \sin n\theta$$

$$R = \Sigma\, R_n^c r^n \cos n\theta + R_n^s r^n \sin n\theta; \quad R_n^c = \Sigma\, R_{nk}^c r^{2k}$$

$$T = \Sigma\, T_n^c r^n \cos n\theta + T_n^s r^n \sin n\theta; \quad R_n^s = \Sigma\, R_{nk}^s r^{2k}.$$

With these definitions R and T are polynomials in x and y. The third and fourth kind of terms have been added in order to have all possible polynomials for x and y. However they seem to be very unlikely to appear in any true situation and we will try to avoid them.

Satisfying condition (3).

a) If D_1, f, g is a solution, CD_1, cf, cg is also a solution, C being any function of r. As the distortions are small enough, one good way to choose C is to request that

$$R_0^c \equiv 1, \text{ and } D_1 = F_0(1 + R_{0,1}^c r^2 + R_{0,2}^2 r^4 + \ldots).$$

b) If one made an infinitesimal rotation of the camera system with respect to the chamber system:

$$\vec{x} \rightarrow \vec{x} + \vec{\omega} \times \vec{x},$$

one finds that this induces the following transformation:

$$x \rightarrow x - \omega_3 y + \frac{x}{D_1}(-\omega_1 y + \omega_2 x) + \ldots$$

$$y \rightarrow y + \omega_3 x + \frac{y}{D_1}(-\omega_1 y + \omega_2 x) + \ldots$$

So by inspection one finds that it should be requested that

$$T_{0,0}^{c} = 0 \quad R_{1,0}^{c} = 0 \quad R_{1,0}^{s} = 0.$$

COLD VERSUS WARM CHAMBER OPTIC

For a same image (x, y) onto the film, according to W. Smart, the light-ray angles with respect to the optical axis are related by:

$$\psi = \theta + A\Delta\sin\theta,$$

where θ is the cold angle and ψ the warm one. Translated in terms of our optics description this means that one has

$$D_1^{\text{warm}} = D_1^{\text{cold}} - A\Delta(D_1^{2\,\text{cold}} + x^2 + y^2)^{1/2}$$

with $A\Delta = 16.113 \times 10^{-5}$.

OPTICS COMPUTER HANDLING

Appendix I gives the coding of the two routines PROPAG, computing D_1 and D_2, and DISTOR, computing f and g; this coding is the one used by HYDRA geometry. Quite analogous coding is also available for LBCG.

Appendix II displays the cards used to read in the data needed by PROPAG and DISTOR.

Description of the title (Appendix II).

Block CAME contains for each camera the six parameters $X_c Y_c Z_c\ \theta_1 \theta_2 \theta_3$. The three blocks OPT1, OPT2, OPT3 refer to the three cameras and contain data to be used by PROPAG and DISTOR. Those blocks are stored in array Q (as is everything else). For each of those three blocks the meaning is the following:

Q(IPOP-1)	1	Type of optics (fisheye)
Q(IPOP)		$F_0 = D_1(0)$
Q(IPOP+1)	$N_2=1$	Number of terms for D_2 expansion
Q(IPOP+2)	0	Last (and only) term of D_2 expansion
Q(IPOP+Q(IPOP+1)+1)	NT	highest symmetry-1

For each symmetry n one has:*

a_{n-1} ("stretch")

b_{n-1}

KM maximum-1 power in r^2

$R^c_{n,KM-1}$

$R^c_{n,KM-2}$

.

.

$R^c_{n,0}$

$R^s_{n,KM-1}$

$R^s_{n,KM-2}$

.

.

$R^s_{n,0}$

$T^c_{n,KM-1}$

.

.

$T^s_{n,0}$

*There is no room assigned for the following terms: $R^c_{1,0}$, $R^s_{1,0}$ and $T^c_{0,k}$.

APPENDIX I

```
SUBROUTINE PROPAG(IPOP,XY)
DIMENSION XY(4)
  ---COMPUTES OPTICS PARAMETER AS FOLLOW:
  D2=2.
  D1=F0*(1.+RCC1*R**2+RC*2*R**4+.........)
  R IS DISTANCE FROM MEASUREMENT TO OPTICAL AXIS/FOCAL LENGH
  FOR AXIAL LIGHT RAYS F0 ,
COMMON /QBITS/ IQDROP,IQMARK,IQGO,IQGONE,IQSYS,IQCRIT                   Q
                        DIMENSION       IQUEST(32), Q(99), IQ(99)       +Q
                        EQUIVALENCE (QUEST,IQUEST), (LQUSER,IQ,Q)       +Q
COMMON //       QUEST(32),LQUSER(7),LQMAIN,LQSYS(16),LQPRIV(7)          +Q
,               LQ1,LQ2,LQ3,LQ4,LQ5,LQ6,LQ7,LQSV,LQAN,LQDW,LQUP         +Q
R2=(XY(1)**2+XY(2)**2)/Q(IPOP)**2
IP=IPOP+Q(IPOP+1)+2
IP=3+IP
N=Q(IP)
A=0.
DO 2 I=1,N
IP=1+IP
A=(A+Q(IP))*R2
XY(3)=-(1.+A)*Q(IPOP)
XY(4)=0.
RETURN
END
SUBROUTINE DISTOR(NVW,XF)
  ---COMPUTES DISTORTIONS AS FOLLOW:
  XF CONTAINS X,Y MEASUREMENT -ROW VALUES ON INPUT
  -DISTORTED VALUES ON OUTPUT
  XF(1)+XF(1)+RADIAL*XF(1)-TANGEANTIEL*XF(2)+STRECH(1)
  XF(2)+XF(2)+RADIAL*XF(2)+TANGEANTIEL*XF(1)+STRECH(2)
  WHERE
  STRECH(1)=SUM/N OF (AN*COS(N*T)+BN*SIN(N*T))*R**N
  STRECH(2)=SUM/N OF (BN*COS(N*T)-AN*SIN(N*T))*R**N
  RADIAL=SUM/N OF (RCN*COS(N*T)+RSN*SIN(N*T))
  TANGEANTIEL=SUM/N OF (TCN*COS(N*T)+TSN*SIN(N*T))
  WITH RCN=SUM/K OF RCNK*R**(2*K+N)  AND ANALOGOUS FOR RSN,TCN,TSN


  T=AZIMUTH OF VECTOR XF
  R=MODULUS OF XF/FOCAL LENGH FOR AXIAL LIGHT RAYS F0

  SOME OF THOSE PRECEDINGS ARE NOT INCLUDED TO PREVENT DEGENERACY
  WITH CAMERA'S POSITION AND ANGLES PARAMETERS. (SEE WRITE-UP)
COMMON/OPTICS/CAME(3,3),RM(9,3),IPOPT(3)
COMMON /QBITS/ IQDROP,IQMARK,IQGO,IQGONE,IQSYS,IQCRIT                   Q
                        DIMENSION       IQUEST(32), Q(99), IQ(99)       +Q
                        EQUIVALENCE (QUEST,IQUEST), (LQUSER,IQ,Q)       +Q
COMMON //       QUEST(32),LQUSER(7),LQMAIN,LQSYS(16),LQPRIV(7)          +Q
+,              LQ1,LQ2,LQ3,LQ4,LQ5,LQ6,LQ7,LQSV,LQAN,LQDW,LQUP         +Q
DIMENSION XF(2),CRT(2) ,XP(2)
DIMENSION XX(2),ST(2)
COMPLEX CC,CP,CX,CS
EQUIVALENCE (CC,CRT),(CP,XP),(CX,XX),(ST,CS)
IPOP=IPOPT(NVW)
IP=IPOP+Q(IPOP+1)+2
NZ=Q(IP)
IF(NZ.LT.1)  RETURN
XX(1)=XF(1)/Q(IPOP)
XX(2)=XF(2)/Q(IPOP)
R2=XX(1)**2+XX(2)**2
CP=CX
ST(1)=Q(1+IP)*XX(1)+Q(2+IP)*XX(2)
ST(2)=Q(2+IP)*XX(1)-Q(1+IP)*XX(2)
IP=3+IP
CRT(1)=1.
CRT(2)=0.
IF(NZ.LT.2)  GO TO 7
IP=IP+Q(IP)+1
DO 3 I=2,NZ
IS=IP
IP=2+IP
N=Q(IP)
IP=1+IP
IF(N.LT.0)  GO TO 2
DO 6 K=1,2
DO 4 M=1,2
A=0.
IF(N.LT.1)  GO TO 5
DO 1 L=1,N
A=(A+Q(IP))*R2
IP=1+IP
IF(K.EQ.1.AND.I.EQ.2)  GO TO 4
A=A+Q(IP)
IP=1+IP
CRT(K)=CRT(K)+A*XP(M)
CONTINUE
CP=CP*CX
ST(1)=ST(1)+Q(IS)*XP(1)+Q(IS+1)*XP(2)
ST(2)=ST(2)+Q(IS+1)*XP(1)-Q(IS)*XP(2)
CONTINUE
CC=CC*CX+CS
XF(1)=CRT(1)*Q(IPOP)
XF(2)=CRT(2)*Q(IPOP)
RETURN
END
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APPENDIX II

```
C*LIST         15 '   CSENSITIVE
C        CONSTANTS SET UP ON 1Ø/18/73 FOR BLOCKS
C        CAMERA,CAMFD,OPT1,2,3,
C         FOR COLD .99706+-.0003 APPLIED  10/4/73.
*      CAME        24  (3F1Ø.Ø/(F1Ø.Ø,3F1Ø.4,3F1Ø.6))
         3         7         1
         1.  -10.7443   -Ø.8744  225.5Ø69     Ø.Ø134     Ø.ØØ69    -Ø.ØØØØ
         2.   95.2544   7Ø.9Ø16  185.6541     Ø.1989     Ø.3445     Ø.ØØØ1
         3.   94.356Ø  -71.7962  185.235Ø     Ø.1919    -Ø.3549    -Ø.ØØØ1
*      FIEL         1   (F1Ø.5)
 .3352
*      CAMF        61   (F1Ø.Ø/3(2F1Ø.Ø/6(F1Ø.Ø,2F1Ø.5/)))
         3
         1         6
         1    3.451Ø3   2.68691
         2    3.39547  -2.64473
         3   -3.76Ø45  -1.99752
         4    2.361Ø4   1.28426
         5    Ø.18Ø8Ø   2.57Ø94
         6   -1.1Ø194  -2.17359
         2         6
         1    3.42596   2.59Ø81
         2    3.36594  -2.67281
         3   -3.83718  -2.Ø3475
         6
         5
         6
         3         6
         1    3.398Ø4   2.69451
         2    3.41883  -2.62292
         3   -3.78676  -2.Ø4223
         4    1.67922   1.3Ø178
         5   -Ø.47697   2.Ø4487
         6   -2.44724  -2.37556
C     CARD NOT READ BUT USEFUL
*      OPT1        35  (F1Ø.Ø/(4E15.8))
         1.
 Ø.36195390E+Ø1 Ø.1ØØØØØØØE+Ø1 Ø.ØØØØØØØØE+ØØ Ø.4ØØØØØØØE+Ø1
-Ø.19780190E-Ø3 Ø.4Ø636242E-Ø4 Ø.4ØØØØØØØE+Ø1 Ø.11155149E-Ø1
 Ø.67231Ø79E-Ø2 Ø.23378706E-Ø1-Ø.28644877E+ØØ-Ø.82380175E-Ø4
-Ø.16431638E-Ø3 Ø.1ØØØØØØØE+Ø1 Ø.13Ø93445E-Ø2 Ø.81152179E-Ø3
 Ø.ØØØØØØØØE+ØØ-Ø.2328239ØE-Ø2 Ø.ØØØØØØØØE+ØØ Ø.27237Ø75E-Ø2
 Ø.42340187E-Ø4-Ø.69456112E-Ø4 Ø.ØØØØØØØØE+ØØ Ø.167ØØ727E-Ø3
-Ø.51847357E-Ø4 Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ
 Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ Ø.5732653ØE-Ø4 Ø.48777377E-Ø3
 Ø.34565512E-Ø3-Ø.2Ø768597E-Ø3
*      OPT2        35   (F1Ø.Ø/(4E15.8))
         1.
 Ø.36284Ø92E+Ø1 Ø.1ØØØØØØØE+Ø1 Ø.ØØØØØØØØE+ØØ Ø.4ØØØØØØØE+Ø1
-Ø.3122Ø957E-Ø3-Ø.27788617E-Ø4 Ø.4ØØØØØØØE+Ø1 Ø.15499338E-Ø1
-Ø.5247Ø119E-Ø3 Ø.3Ø233511E-Ø1-Ø.28662142E+ØØ Ø.ØØØØØØØØE+ØØ
 Ø.ØØØØØØØØE+ØØ Ø.1ØØØØØØØE+Ø1 Ø.ØØØØØØØØE+ØØ-Ø.1Ø692821E-Ø2
-Ø.33813337E-Ø3 Ø.21627624E-Ø2 Ø.ØØØØØØØØE+ØØ Ø.73578499E-Ø3
 Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ
 Ø.83864128E-Ø4 Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ
 Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ Ø.52144332E-Ø4 Ø.64219579E-Ø4
-Ø.49852599E-Ø4 Ø.1Ø53256ØE-Ø4
*      OPT3        35   (F1Ø.Ø/(4E15.8))
         1.
 Ø.36261163E+Ø1 Ø.1ØØØØØØØE+Ø1 Ø.ØØØØØØØØE+ØØ Ø.4ØØØØØØØE+Ø1


-Ø.223ØØ236E-Ø3-Ø.731Ø4563E-Ø4 Ø.4ØØØØØØØE+Ø1 Ø.15269974E-Ø1
-Ø.19741437E-Ø3 Ø.3ØØ21157E-Ø1-Ø.28699220E+ØØ Ø.ØØØØØØØØE+ØØ
 Ø.ØØØØØØØØE+ØØ Ø.1ØØØØØØØE+Ø1 Ø.3Ø489Ø17E-Ø3 Ø.1954255ØE-Ø3
-Ø.1365308ØE-Ø3-Ø.52ØØ5847E-Ø3 Ø.ØØØØØØØØE+ØØ Ø.3249572ØE-Ø3
 Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ
 Ø.1834272ØE-Ø4 Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ
 Ø.ØØØØØØØØE+ØØ Ø.ØØØØØØØØE+ØØ Ø.1269Ø959E-Ø4 Ø.ØØØØØØØØE+ØØ
 Ø.ØØØØØØØØE+ØØ-Ø.1257Ø7Ø7E-Ø4
```

APPENDIX III

CAMERA SYSTEM TO CHAMBER SYSTEM TRANSFORMATION

The rotation between camera system and chamber system is made with a rotation matrix in the following way:

$$X^i_{chamber} = X^i_c + R_{ij}\, x^j_{camera},$$

R being the following matrix:

$$\begin{pmatrix} \cos\theta_1\cos\theta_3 & ,\ \cos\theta_1\sin\theta_3 & ,\ \sin\theta_1 \\ -\cos\theta_2\sin\theta_3-\sin\theta_1\sin\theta_2\cos\theta_3, & \cos\theta_2\cos\theta_3-\sin\theta_1\sin\theta_2\sin\theta_3, & \cos\theta_1\sin\theta_2 \\ \sin\theta_2\sin\theta_3-\sin\theta_1\cos\theta_2\cos\theta_3, & -\sin\theta_2\cos\theta_3-\sin\theta_1\cos\theta_2\sin\theta_3, & \cos\theta_1\cos\theta_2 \end{pmatrix}$$

As an example, the vector $\begin{matrix}0\\0\\1\end{matrix}$ optical axis in the camera system will be transformed in the chamber system as $\begin{matrix}\sin\theta_1\\ \cos\theta_1\sin\theta_2\\ \cos\theta_1\cos\theta_2\end{matrix}$.